## Unified Engineering Problem Set Week 13 Fall, 2007

Lectures: M18, M19, M20 Units: M2.4 (part), M3.1

**M13.1** (*15 points*) In Problem M10.1, a structure made of a uni-directional composite material that has all its fibers oriented at an angle of 25° relative to the loading axes was considered. The structural loading produced the following state of plane stress in the loading axis system:

$$\sigma_{11} = 15 \text{ ksi}$$
  

$$\sigma_{22} = -10 \text{ ksi}$$
  

$$\sigma_{12} = -5 \text{ ksi}$$

This situation is illustrated in the accompanying figure.



Work was done to find the stress state along the "composite fiber axes". These results are:

$$\sigma'_{11} = 6.7 \text{ ksi}$$
  
 $\sigma'_{22} = -1.7 \text{ ksi}$   
 $\sigma'_{12} = -12.8 \text{ ksi}$ 

where the ' indicates the stresses in the "composite fiber axes".

We now need to extend our analysis of this situation. All work should be done relative to the original loading axes.

- (a) Determine the principal stresses and the associated directions.
- (b) Find the maximum shear stresses and their associated planes.
- (c) Indicate how the principal stresses, the maximum shear stresses, and their associated directions change as the fiber angle ( $\theta = 25^{\circ}$ ) changes direction.
- (d) Draw the Mohr's circle for the base loading situation and check the answers obtained in the original problem (stresses in the composite fiber axes), and the answers to parts (a) and (b). Clearly indicate the physical/geometrical aspects of the circle that correspond to these answers.
- **M13.2** (7.5 *points*) For the following cases, briefly (in one or two sentences) state the primary functional requirement that has to be met. Indicate the associated loads (e.g. tension, compression, shear, impact, cyclic, thermal, electrical), and list the five material properties that you think will be most relevant to meeting this requirement (confine your choices to the list given in Table 1.1 of *Ashby and Jones* or other properties discussed in class). Indicate for each property whether it should have a *high, medium,* or *low* value.
  - (a) Cable used in overhead cranes
  - (b) Components of a truss used in a bridge
  - (c) Kitchen countertop
  - (d) Combustion chamber lining of a jet engine
  - (e) Components of a space truss
  - (f) Leading edge of the space shuttle

- **M13.3** (7.5 *points*) A structural frame is to be made of a number of structural members, all of the same cross-sectional shape and the same given length. The cross-section is a square box with a constant inner wall length of a, and a thickness that is constant for a given design but can change dependent upon the material utilized. Each member must carry a constant load, in either tension or compression, of no greater a magnitude than P. The key design criteria at this point are that the members are to deform as little as possible and weigh as little as possible. Cost is also a consideration. The design variables are the wall thickness of the member and the material used to make the member.
  - (a) Determine the figure(s) of merit that is/are pertinent in this case.
  - (b) For the materials listed in the accompanying table, indicate which you would choose for the structural member depending upon which of the three design considerations are most important: minimization of deformation, minimization of weight, minimization of cost. Be sure to clearly explain your reasoning. Use figures as appropriate.

Material	<b>Density</b> [lb/in <sup>3</sup> ]	<b>Modulus</b> [Msi]	Strain Limit , %	Acquisition Cost, [\$/lb]
Aluminum alloy (2000 series)	0.101	10.5	0.58	6.25
Carbon fiber Composite	0.054	24.2	0.50	85.00
Silicon Carbide	0.108	60.5	2.20	146.00
Steel (low carbon)	0.285	29.0	0.60	1.35
Titanium alloy (TI-6Al-4V)	0.162	16.0	0.73	25.10
Wood	0.022	1.81	0.35	0.98